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Addendum to "Best Polynomial Approximation to Certain Entire Functions"

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The notation and definitions herein are the same as those in Ref. [2].

THEOREM 17. *Let $f(z) = \sum_{k=0}^{\infty} a_k z^k$ be an entire function of perfectly regular growth (ρ, τ) with real coefficients. Then*

$$\lim_{n\to\infty} 2^\rho n E_n^{\rho/n} = \rho e \tau. \tag{43}$$

Proof. Since $f(z)$ is an entire function of order $\rho (0 < \rho < \infty)$ and type τ, therefore [1, Theorem 3]

$$\limsup_{n\to\infty} 2^\rho n E_n^{\rho/n} = \rho e \tau. \tag{44}$$

On the other hand [2, p. 105] there exists a strictly increasing sequence $\{n_p\}_1^\infty$ of positive integers such that

$$\lim_{p\to\infty} \frac{n_{p+1}}{n_p} = 1 \tag{45}$$

and

$$\lim_{p\to\infty} n_p E_{n_p}^{\rho/n_p} = \rho e \tau 2^{-\rho}. \tag{46}$$

Clearly

$$E_0 \geqslant E_1 \geqslant E_2 \geqslant \cdots \geqslant E_{n-1} \geqslant E_n \to 0. \tag{47}$$

Set

$$n_p \leqslant n < n_{p+1}; \tag{48}$$

then from (47) and (48) we get

$$\frac{n}{\rho e} E_n^{\rho/n} \geqslant \frac{n_p}{n_{p+1}} \frac{n_{p+1}}{\rho e} E_{n_{p+1}}^{(\rho/n_{p+1})(n_{p+1}/n_p)}. \tag{49}$$

From (45), (46) and (49), we obtain by letting $p \to \infty$,

$$\lim_{n\to\infty} \inf \frac{n}{\rho e} E_n^{\rho/n} \geqslant \tau 2^{-\rho}. \tag{50}$$

The required result (43) follows from (44) and (50).

We take this opportunity to correct misprints in [2].

Page	Line	Read	For
99	11	$\lim_{\mu\to\infty} \frac{2^{n_\mu} E_{n_\mu}(f)}{\lvert a_{n_{\mu+1}} \rvert} = 1$	(11)
105	2	growth (ρ, τ)	growth
105	7	lim sup	lim
106	22	$\lim \genfrac{}{}{0pt}{}{\sup}{\inf} (\ ) = \lim \genfrac{}{}{0pt}{}{\sup}{\inf} (\ ) = \genfrac{}{}{0pt}{}{\tau}{\omega}$	(28)
106	24	$\lim \genfrac{}{}{0pt}{}{\sup}{\inf} (\ ) = \lim \genfrac{}{}{0pt}{}{\sup}{\inf} (\ ) = \genfrac{}{}{0pt}{}{\tau(k)}{\omega(k)}$	(29)
107	19	$k = 1$ and $\rho = 1$	$k = 1$
107	22	exponential type	type
108	11	Z^k	Z_k
108	19	$\left(\frac{\pi}{4}\right)^{n+1} \frac{1}{2^n(n+1)!}$	$\frac{1}{2^n(n+1)!}$

References

1. A. R. Reddy, Approximation of an entire function, *J. Approximation Theory* **3** (1970), 128–137.
2. A. R. Reddy, Best polynomial approximation to certain entire functions, *J. Approximation Theory* **5** (1972), 97–112.